Annotated bibliography of strict finitism

Overview

I. Sources of historical importance

- Paul BERNAYS: "Platonism in Mathematics". Url: http://www.phil.cmu.edu/projects/bernays/Pdf/platonism.pdf. As mentioned on the title page: "Sur le platonisme dans les mathématiques. Lecture delivered June 18, 1934, in the cycle of Conférences internationales des Sciences mathématiques organized by the University of Geneva, in the series on Mathematical Logic. Translation by: Charles Parsons". No published counterpart at present.

To be absolutely clear: Bernays is not to be considered a strict finitist. In the context of a critical analysis of intuitionism he does ask the question whether an intuitionist should not go all the way and also question whether all arithmetical operations are total. A quote: "Intuitionism makes no allowance for the possibility that, for very large numbers, the operations required by the recursive method of constructing numbers can cease to have a concrete meaning. From two integers k, l one passes immediately to k^l; this process leads in a few steps to numbers which are far larger than any occurring in experience, e.g., 67^(257^729)). Intuitionism, like ordinary mathematics, claims that this number can be represented by an Arabic numeral. Could not one press further the criticism which intuitionism makes of existential assertions and raise the question: What does it mean to claim the existence of an Arabic numeral for the foregoing number, since in practice we are not in a position to obtain it?" (I have made a slight adaptation for the power notation). He immediately continues by making clear that this is not his viewpoint but the thought itself is quite interesting and important: a "true" intuitionist should be a strict finitist.

- David VAN DANTZIG: "Is $10^{10^{10}}$ a finite number?". *Dialectica*, vol. 9, nrs. 3/4, 1956, pp. 273-278.

One of the pioneering papers in strict finitism. The negative answer to the question in the title makes the author's position quite clear. Crucial idea: different number systems that cannot be put together into a single framework. Similar to Brian Rotman and Marc Burgin. There are not that many books and papers that make reference to this paper of Van Dantzig. A notable exception is G. I. Ruzavin, *Die Natur der mathematischen Erkenntnis*, Berlin: Akademie Verlag, 1977 (translated from Russian, original, 1968, no English translation). It is quite interesting to see Van Dantzig's name appear in the same paragraph as Friedrich Engels.

- Ludwig WITTGENSTEIN: *Remarks on the Foundations of Mathematics*. (G.H. von Wright, R. Rhees and G.E.M. Anscombe (eds.) – translated by G.E.M Anscombe.) Oxford: Basil Blackwell, 1956 [1978].

All too often, if strict finitism is mentioned, one of "the founding fathers" is considered to be Ludwig Wittgenstein and, more specifically, the later Wittgenstein, author of the book mentioned above. As is well known, reading Wittgenstein is very often quite a challenge and the *Remarks* are no different in that respect. Hence there exists a rather extensive literature on this particular topic. A very nice and nuanced overview is to be found in:

- Victor RODYCH: "Wittgenstein's Philosophy of Mathematics". *The Stanford Encyclopedia of Philosophy* (Spring 2018 Edition), Edward N. Zalta (ed.), url = <https://plato.stanford.edu/archives/spr2018/entries/wittgenstein-mathematics/>.

Note that this lemma covers both the early (*Tractatus*) and later Wittgenstein. Let me just mention here a few items of importance. It starts with:

- Charles F. KIELKOPF: *Strict Finitism. An Examination of Ludwig Wittgenstein's* Remarks on the Foundations of Mathematics. The Hague: Mouton, 1970.

Up to my knowledge one of the first discussions of Ludwig Wittgenstein contribution to the philosophy of mathematics. Often it is claimed that his position was some form of strict finitism. Later on, many authors (see further) have disputed this claim and rather emphasize the 'agnostic' (or some other) attitude of Wittgenstein. It remains nevertheless quite interesting to see how an author, *in casu* Wittgenstein, can be read in a particular way.

- A.S. YESSENIN-VOLPIN: "Le programme ultra-intuitioniste des fondements des mathématiques". In: *Infinitistic Methods, Proceedings Symposium on Foundations of Mathematics*. Oxford: Pergamon Press, 1961, pp. 201-223.
- A.S. YESSENIN-VOLPIN: "The ultra-intuitionistic criticism and the antitraditional program for foundations of mathematics". In: Kino, Myhill & Vesley (eds.): *Intuitionism & proof theory*. Amsterdam: North-Holland, 1970, pp. 3-45.
- A.S. YESSENIN-VOLPIN: "About infinity, finiteness and finitization". In F. Richman (ed.), *Constructive mathematics*. Berlin-New York: Springer, 1981, pp. 274-313.

These are the two 'famous' papers of Alexander Yessenin-Volpin, considered by many to be the 'founding father' of strict finitism. The story however is more complex than that. He was not particularly interested in strict finitism *per se*, but in relation to a(n unorthodox) consistency proof for ZFC. Quite a different thing!

Apart from the few papers in English, most of his work is published in Russian. I mention some of these for completeness' sake:
1. A.S. Yessenin-Volpin: "Analysis of the potential feasibility" ("Logicheskije Issledovanija"), AN SSSR, Moskva, 1959, pp. 218–262 (in Russian).
2. A.S. Yessenin-Volpin: "On the Grounding of Set Theory". In: *Application of Logic in Science and Technology*, Moscow, 1960, pp. 22–118 (in Russian).
Three very curious items should be added to this material:
1. In L.J. Cohen, J. Łoś, H. Pfeiffer & K.-P. Podewski (eds.): *Logic, Methodology and Philosophy of Science VI* (Amsterdam: North-Holland, 1982) on p. 843 in the programme of the conference, held in Hannover from 22 to 29 August 1979, in the section on proof theory and foundations of mathematics a contributed paper is mentioned by Volpin, titled "On an Explanation of an Anti-Traditional Paradox". The book itself does not contain the paper itself nor an abstract and I have been unable to trace it down.
2. Even stranger is the paper, written jointly with Christer Hennix, "Beware of the Gödel-Wette Paradox!" that appeared on the ArXiv.org website in 2002, submitted in 2001 (arXiv:math/0110094v2 [math.LO]). It still continues more or less the same research program but now it becomes even more difficult to understand what is happening. The abstract states that "This paper gives a counter-example to the impossibility, by Gödel's second incompleteness theorem, of proving a formula expressing the consistency of arithmetic in a fragment of arithmetic on the assumption that the latter be consistent." (p.1) Although the association with the work of Eduard Wette is denied, this is once again not the best choice to make for a *compagnon de route*. It is worth mentioning that in the bibliography of this paper another paper by Yessenin-Volpin is mentioned: *A Completeness Proof for Ari-like Systems* (in preparation). So far no trace has been found of this paper.
3. Not of tremendous importance but rather surprising to meet Yessenin-Volpin here. In the *Notices of the AMS*, 64(5), 2017, pp. 504-507, Judith Roitman, mathematician and poet, reviews two books, one of which is *Gallery of the Infinite* by Richard Evan Schwartz (published by the AMS in 2016) and she remarks on page 506 that: "And also some nonbasics: the mathematical universe based on the empty set ("from this point of view, numbers are just organized emptiness"); brief mention of the axioms of ZF; and speculation about how maybe none of this makes sense because there might be a flaw in the axiom system (accompanied by a wild-eyed figure looking a little like a beardless Esenin-Volpin (the resemblance might just be my imagination)." There is a footnote attached to his name and it says: "He denied the existence not only of infinite sets but of very, very large ones.". Indeed!
A biography is to be found at: http://yesseninvolpin.net/bio.html (on the basis of interviews by John Karas in 2002).
Another small addition of historical interest: in the fourth volume of the collected works of Kurt Gödel there are letters making reference to Yessenin-Volpin in his correspondence with Paul Bernays (perhaps this should no longer amaze us, given the entry above). Although Gödel did not particularly like the way Yessenin-Volpin dealt with ultra-intuitionism, the topic itself did. The details are to be found in: Feferman, Solomon; John W. Dawson, Jr.;

Warren Goldfarb; Charles Parsons & Wilfried Sieg (eds.): *Kurt Gödel. Collected Works. Volume IV: Correspondence A-G*. Oxford: Oxford University Press, 2003, pp. 57-59.

- Paolo MANCOSU: "Harvard 1940–1941: Tarski, Carnap and Quine on a Finitistic Language of Mathematics for Science". *History and Philosophy of Logic*, **26**, 2005, pp. 327–357.
- Greg FROST-ARNOLD: *Carnap, Tarski, and Quine at Harvard: Conversations on Logic, Mathematics, and Science*. Chicago, IL: Open Court, 2013.

A discovery to fundamentally change the history of strict finitism. I had the idea that David van Dantzig was the best 'candidate' to qualify as founding father and that it was wise to be careful with Yessenin-Volpin and Wittgenstein. But now it appears that the starting date might just as well be 1940-1941. Both Mancosu and Frost-Arnold have studied this miraculous year in Harvard where Tarski, Carnap and Quine discusses many things, among them the possibility of a finite language for mathematics and for science. Although it must be added straightaway that such an approach is not necessarily to be interpreted as strict finitism. It can also be seen as a common core, a first step in a further construction. But, truly interesting is the fact that Carnap started work on this finite language and quite amazingly, he listed three possibilities for a finite arithmetic and these are precisely the most often quoted proposals in strict finitism studies. In a nutshell: (a) a fixed-point model, where the successor of a specific number is that number itself, $s(n) = n$, (b) a cyclic model, where for some number n, $s(n) = 0$, and (c) a repetitive model, that is, there is a number such that $s(n) = 0'$, where 0' starts a new sequence of natural numbers. However! After that year not much happened, except that Quine and Nelson Goodman wrote a paper on nominalism and arithmetic that is a continuation of the work done in that year. [Nelson GOODMAN and W. V. QUINE: "Steps Toward a Constructive Nominalism". *The Journal of Symbolic Logic*, **12**(4), 1947, pp. 105-122.] But the approach outlined in that paper is no longer a form of strict finitism, as it allows for the potential infinite. If one is looking for a 'quick' introduction to these fascinating discussions, read the following book review: Gary EBBS: "Carnap, Tarski, and Quine at Harvard: Conversations on Logic, Mathematics, and Science". *History and Philosophy of Logic*, **36**(2), 2015, pp. 181-188.

- Ernst WELTI: *Die Philosophie des Strikten Finitismus. Entwicklungtheoretische und mathematische Untersuchungen über Unendlichkeitsbegriffe in Ideengeschichte und heutiger Mathematik*. Bern: Peter Lang, 1986.

All of the above are original sources including commentators on those sources. If the question is asked whether historical-philosophical surveys exist of strict finitism, the answer is nearly no, because there is one outstanding exception, namely the book of Ernst Welti. Originally a doctoral thesis under the supervision of E. Engeler and, perhaps somewhat unexpectedly, Paul Feyerabend. As far as I know, no English translation exist. It also urgently needs an update. But then it is good to know that a lot of 'ground work' has already been done.

II. Sources referring to mathematics itself

II.1. Arithmetic

- Mirco A. MANNUCCI & Rose M. CHERUBIN: "Model Theory of Ultrafinitism I: Fuzzy Initial Segments of Arithmetic (Preliminary Draft)". Url: https://arxiv.org/abs/cs/0611100v1, 2006 (consulted 29 August 2019).

The abstract is an excellent summary: "This article is the first of an intended series of works on the model theory of Ultrafinitism. It is roughly divided into two parts. The first one addresses some of the issues related to ultrafinitistic programs, as well as some of the core ideas proposed thus far. The second part of the paper presents a model of ultrafinitistic arithmetics based on the notion of fuzzy initial segments of the standard natural numbers series. We also introduce a proof theory and a semantics for ultrafinitism through which feasibly consistent theories can be treated on the same footing as their classically consistent counterparts. We conclude with a brief sketch of a foundational program, that aims at reproducing the transfinite within the finite realm." Unfortunately, at the present moment this preliminary draft is the only outcome of their research. Further note that this paper could just as well have been classified under history of strict finitism (as a large part of the paper outlines a brief history of, what they call, ultrafinitism) and/or under vague approaches for that is their main approach.

- Francesco BERTO: *There's Something About Gödel. The Complete Guide to the Incompleteness Theorem*. Oxford: Wiley-Blackwell, 2009.

This is the English of the Italian original published in 2008. Berto is not himself a strict finitist but on pages 210-212 both the work of Graham Priest and myself, in relation to paraconsistent number theory is mentioned. So authors of overviews such as this one start to pick up (occasionally) on strict finitism. Berto has been working on these themes more extensively, witness this earlier publication: Francesco BERTO: "The Gödel Paradox and Wittgenstein's Reasons". Philosophia Mathematica (III), **17**, 2009, pp. 208–219.

- M. BURGIN: "Introduction to Projective Arithmetics". Url: http://arxiv.org/ftp/arxiv/papers/1010/1010.3287.pdf, 2010. (This is an Arxiv-paper, as yet without a published counterpart.)

The paper discusses *non-Diophantine arithmetics*, that come in two variants: projective and dual arithmetics. This paper deals with the former type. (Actually from the strict finitist perspective, the latter type is less interesting as all these models are infinite). The basic schema is to have a function f back and forth between N, the "full" natural numbers, and some subset A of N in such a way that addition, multiplication, and order can be defined. If the function is such that numbers "come closer" to one another, say $f(n) = n^2$, then a number L can be reached such that L = L + 1. And that comes close to strict finitism. The similarities with Brian Rotman's approach are clear. Although he talks about non-Euclidean arithmetics. Burgin does draw the parallel but insists on a different name.

Quote: "The most extreme assertion that there is only a finite quantity of natural numbers was suggested by Yesenin-Volpin (1960), who developed a mathematical direction called ultraintuitionism and took this assertion as one of the central postulates of ultraintuitionism. Other authors also considered arithmetics with a finite number of numbers, claiming that these arithmetics are inconsistent (cf., for example, (Meyer and Mortensen, 1984; Van Bendegem, 1994; Priest, 1997; 2000; Rosinger, 2008))." (p. 5)

Other publications: there is a somewhat similar problem here as with Yessenin-Volpin. Most of the other relevant publications of this author are either in Russian or in Ukranian. References can be found on the author's website: http://www.math.ucla.edu/~mburgin/.

- Roy SORENSEN: *A Cabinet of Philosophical Curiosities. A Collection of Puzzles, Oddities, Riddles and Dilemmas*. London: Profile Books, 2016.

This book by the well-known logician and philosopher Roy Sorensen (author of the famous *Blindspots*) contains a short passage (on pp. 214-215, under the beautiful title "Why one is the loneliest number") that makes reference to mathematician Oskar Perron who proposed the following argument to refute strict finitism (this is not exactly the version of Sorensen but it is closer to the original version, I believe):

> "Suppose that N is the largest natural positive number. Since for $N > 1$, $N^2 > N$ and $N^2 > N$ is impossible since N is the largest number, it follows that $N = 1$. Hence 1 is the largest number."

Actually, this is not a bad argument at all although it seems to be dismissed as a fallacy. It is easy of course to see the loophole in the "proof" but the question remains what to do with statements such as N^2, when N is the largest number around. Should not $N^2 = N$? But, if so and if the usual rules apply then $N^2 – N = 0$ or $N*(N – 1) = 0$, hence either $N = 0$ or $N = 1$. So the answer must be that the usual rules do not apply which is an interesting insight.

- Timothy Y. CHOW: "The Consistency of Arithmetic". *The Mathematical Intelligencer*, **41**(1), 2019, pp. 22-30.

Obviously consistency and inconsistency are important topics to discuss within the framework of strict finitism. In several places, e.g., in relation to inconsistent/paraconsistent mathematics and logics, the subject has already been mentioned. In the case of Timothy Chow's paper – and, as in so many cases, I hasten myself to state clearly that he is not a strict finitist! – the general question of consistency is presented and discussed and, therefore, also mention is made of the finite case. Roughly, what we are talking about here is (fragments of) Peano Arithmetic and the provability of such statements as "A proof of a contradiction will require at least n steps (or symbols)". The philosophical importance is clear: if n is sufficiently large then the strict finitist, labelled, by the way, *Ulphia* by Chow, need not have any worries as a contradiction will not turn up, given sufficiently limited means and resources. Equally interesting is to see whether or not such statements are provable in weaker systems than PA. Chow makes reference to an indeed interesting paper by Pavel Pudlák [Pavel PUDLAK: “Incompleteness in the finite domain”. *Arxiv*:1601.01487v2 [math.LO], 18 May 2017]. I only mention it in order to avoid misunderstandings, but Pudlák is not a strict finitist.

II.2. Geometry

A major part of the literature under this heading deals with classical geometry directly, in the sense that one looks for strict finitist simulations or approximations to the classical approach to show that both are viable. See my lemma in the *Stanford Encyclopedia of Philosophy* (reference given below in section VI) for a number of references. Most of the references listed in this section do not appear in that lemma. All that being said, it would be a major omission not to mention *mereology* as a form of pre-geometry. Although I did not encounter papers that make a lot of fuss about discrete mereology, it is an interesting approach as the basic theory does not make the distinction between finite, discrete and infinite. It is something that can be added later on but no so initially. In a few words: mereology is the study of the part-whole relationship. Thus Pxy meaning “x is a part of y” is taken as a primitive. There can be ‘atoms’, defined by $Ax =_{df} \neg(\exists y)Pyx$, literally stating that an atom has no parts. Note that this does not say anything about their extension. Atoms are not necessarily points in the classical geometrical sense. Mereology has the additional advantage that it almost invites us to have a look at the work of Alfred North Whitehead who developed a theory of point-less geometry. I will not discuss this topic here any further but invite the reader to consult: Varzi, Achille: "Mereology". *The Stanford Encyclopedia of Philosophy* (Spring 2016 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/spr2016/entries/mereology/>.

- Donald L. REISLER & Nicholas M. SMITH: *Geometry Over a Finite Field*. Fort Belvoir, VA: Defense Technical Information Center, 1969. (Full text: https://apps.dtic.mil/dtic/tr/fulltext/u2/714115.pdf).

A quite remarkable report, rather extensive. One of the few examples, as far as I know, where the work of Kustaanheimo and Jarnefelt (see elsewhere in this bibliography, also mentioned in the *Stanford Encyclopedia* lemma) is further developed. (Another example is the Belgian philosophers Leo Apostel.) The goals remains the same: replace the field of the reals by a finite field and check whether a geometry results that is comparable to Euclidean continuous geometry.

- Tim MAUDLIN: *New Foundations for Physical Geometry. The Theory of Linear Structures*. Oxford: Oxford University Press, 2014.

The author proposes another road to approach the underlying mathematics of physical theories. Instead of topology the theory of linear structures is proposed. (This book is actually the first part of a two-volume project). Although the author himself is not involved with strict finitism, the interesting thing about this approach is that it applies just as well to discrete as to continuous geometries. In that sense, it provides an alternative route to axiomatisations of discrete geometry. I have listed this book here because only in volume two will the physical applications become clear. Here the focus is on the mathematical background.

II.3. Analysis (and more)

- Jan MYCIELSKI: "Analysis without actual infinity". *Journal of Symbolic Logic*, 46, 1981, pp. 625-633.

This is a true "classic" in the reformulation of analysis without reference to actual infinity. Again, one should be warned that Mycielski is not a strict finitist but, somewhat similar to Graham Priest, is actually interested in constructing finite models for arithmetic without excluding the infinite models. But given such a finite model with a largest number N, then the inverses 1/M of suitable numbers M, M >> N, can serve as a kind of infinitesimals and thus a form of analysis can be developed. As far as I know, this paper remains something special in the oeuvre of Mycielski.

- Doron ZEILBERGER: " 'Real' analysis is a degenerate case of discrete analysis". This short paper is a transcript of a plenary talk delivered at the *International Conference on Difference Equations and Applications* (ICDEA 2001), Augsburg, Germany, Aug. 1, 2001, 9:00-10:00 a.m. (This truly detailed reference is given in the paper itself and, as this material has not been published in paper form, it can be found on: http://sites.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/real.html, consulted August 15, 2019.

Here at least is a mathematician who clearly claims to be a strict finitist, although ultrafinitist is his own choice of words. It is rather intriguing that Zeilberger is, on the one hand a 'standard' mathematician (and recognised as such) but, on the other hand, on his website in opinion pieces and comments and criticisms, he defends the thesis that talk about the existence of infinity is total nonsense (as the expression goes, it is so wrong, it is not even false). As he says himself: this double appearance is in order to avoid that he would be seen as a 'crackpot'. The paper listed here is one of the few papers where he presents a more detailed outline of ultrafinitism. The reason why there are so few is, in his own words, "because the task is too trivial to do"). What is interesting about his position is this:

> "Myself, I don't care so much about the natural world. I am a platonist, and I believe that finite integers, finite sets of finite integers, and all finite combinatorial structures have an existence of their own, regardless of humans (or computers). I also believe that symbols have an independent existence. What is completely meaningless is any kind of infinite, actual or potential." (page 8)

In an odd way, this view is rather close the David Hilbert's position, as outlined in, e.g., "On the infinite" ("Ueber das Unendliche"). What Hilbert called 'ideal' elements are here referred to as symbols.

- Victor KAC & Pokman CHEUNG: *Quantum Calculus*. New York: Springer, 2002.

A quite intriguing book. Two discrete versions of standard analysis are being developed in great detail, labelled q-calculus and h-calculus. In the former case, the derivative of a function f is defined as $(f(qx) – f(x))/(q – 1)x$ and in the latter case as $(f(x+h) – f(x))/h$. If $q \to 1$ and $h \to 0$ both definitions coincide of course with the classical derivative. Although not an extensive text, a bit more than a hundred pages, it does contain a full-fledged formulation of (both versions of a) discrete calculus, not using the classical framework in the background. Note that, once again, these two mathematicians are interested in the topic for its intrinsic mathematical interest, not because they are strict finitists (at least, it is not acknowledged anywhere in the book).

III. Sources referring to physical theories

- Philip C. JACKSON, Jr.: "Design for a Computer Field Interaction Simulation System for Relativistic, Discrete Unified Field Theories". *Speculations in Science and Technology*, **3**(1), 1980, pp. 29-40.

An early example of "discretizing" continuous models in order to be implemented on computers. This is not strict finitism, of course, but it does show that discrete models can approximate continuous ones as close as possible. Hence the choice of working with the one or the other becomes both a computational matter and a (set of) philosophical question(s) to be addressed. It is to be noted that the models presented are fairly simple and serve more as an argument rather than a demonstration.

- Helge KRAGH & Bruno CARAZZA: "From Time Atoms to Space-Time Quantization: the Idea of Discrete Time, ca 1925-1936". *Studies in the History and the Philosophy of Science*, **25**(3), 1994, pp. 437-462.

A quite important paper from the historical perspective. Many references to sometimes completely forgotten authors such as Georgii Pokrowski, Gottfried Beck, Arthur Ruark, Henry Flint, and Ludwik Silberstein. But of course Arthur Eddington is present as well. Do not expect to get fully worked out theories but rather sketches and ideas about discrete time (and sometimes discrete space) based on the young quantum theory of those days, leading sometimes to, what the authors call, "quantum numerology".

- David FINKELSTEIN: "Finite Physics", in Rolf Herken (ed.), *The Universal Turing Machine. A Half-Century Survey*. New York: Springer, 1995 (2nd edition), pp. 323-347.

It is perhaps a bit generous to include this particular paper of Finkelstein but there is very nice passage where a (not altogether truly original as it has been repeated a number of times, namely: "In such a topology, the usual continuum of zero-size points is only a workable approximation to a finite but immense network of cells of some nonzero size set by a physical constant *H* with the units of time, and the operative principle of locality is that of connection among these cells."

- E. B. DAVIES: "Building Infinite Machines". *The British Journal for the Philosophy of Science*, **52**(4), 2001, pp. 671-682.

A very nice paper that presents a detailed description on how to build an infinite machine (= a machine capable of performing an infinite number of finite tasks in a finite time) in a universe, similar to ours (i.e., obeying the same Newtonian laws, so no quantum mechanics involved), but assuming that matter is continuous and can therefore be used to make machines that become smaller and smaller (though all finite of course). This paper is obviously no direct contribution to strict finitism, but it is definitely related: imagine that such machines were somehow constructible in our universe then that would constitute a serious challenge to strict finitist approaches. This paper situates itself in the broader discussion of supertasks and similar problems. See other entries in this bibliography.

- Andras KORNAI: "Explicit Finitism". *International Journal of Theoretical Physics* **42**, 2, 2003, pp. 301-307.

This paper introduces the J-machine, a sort of almost real computer that incorporates (in a symbolic way) all of the real numbers. The interest of the paper is in the explicit calculation of the size of such a machine and what computations it can handle. Quote:

> "The standard mathematical tools of investigating complexity lack the required resolving power: even the lowest Turing degree lumps together problems which are solvable, such as finding the number of digits in $E^*(6)$, and problems which are not, such as finding the number of digits in $E^*(7)$." (Note: $E^*(m)$ stands for an exponential tower of m 2s).

The paper itself refers to:

- Seth LLOYD: "Computational capacity of the universe". *Physical Review Letters* 88, 2003, pp. 237901-1 – 237901-4.

- Vincent ARDOUREL: "La physique dans la recherche en mathématiques constructives." *Philosophia Scientiae*, vol. **16**(1), 2012, pp. 183-208.
- Vincent ARDOUREL: "A Discrete Solution for the Paradox of Achilles and the Tortoise". *Synthese*, **192**(9), 2015, pp. 2843-2861.
- Vincent ARDOUREL: *Du calcul sur ordinateur à la mécanique discrète*. Paris: Vrin, 2017 (to appear) (Collection Mathesis).

Although one should be careful to label Ardourel a strict finitist (as his interest is in the computational and constructive part of physical theories), the paper on Zeno is quite interesting. The book to appear in 2017 is a book version of his PhD.

- Tobias FRITZ: "Velocity polytopes of periodic graphs and a no-go theorem for digital physics". *Discrete Mathematics* 313, 2013, pp. 1289-1301.

Paper is discussed in my entry "Finitism in Geometry", see http://plato.stanford.edu/entries/geometry-finitism/. Let me remark here that this paper shows mainly a no-go result and as such it is instructive to see what roads should not be further explored.

- John C. AMSON & Louis H. KAUFFMAN (eds.): *Scientific Essays in Honor of H. Pierre Noyes on the Occasion of His 90th Birthday*. Singapore: World Scientific, 2014.

This collection of papers brings together a number of contributions of authors who, to a larger or smaller extent, have been member of the *Alternative Natural Philosophy Association* (ANPA). The ANPA tried to develop an alternative discrete form of physics, starting from the so-called "combinatorial hierarchy", developed by Fredrick Parker-Rhodes. It is also sometimes referred to as "bit-string" physics. Especially in the work of Pierre Noyes, there is a particular interest in calculating physical constants, starting from first principles, thus providing a strong argument for the precise values of those constants. Some more information can be found in the *Stanford Encyclopedia* lemma on finitism in geometry (http://plato.stanford.edu/entries/geometry-finitism/).

- David T. CROUSE: "The lattice world, quantum foam and the universe as a metamaterial. The use of black holes as hyper-resolving microscopes to probe the fine structure of space". *Appl. Phys. A*, 122, 2016, pp. 472 (7 pages).
- David CROUSE & Joseph SKUFCA: "Relativistic Time Dilation and Length Contraction in Discrete Space-Time using a Modified Distance Formula". https://arxiv.org/abs/1706.00693 (uploaded 1 June 2017).

The title of the first paper precisely says what it is about. The second paper is quite interesting because it first brings together and (more or less) unifies existing proposals for a discrete distance formula but secondly it shows how time dilation and length contraction can be given an explanation in such a framework. Especially the connection with special relativity is important because the true challenge for a strict finitist view lies there (and, of course, in quantum mechanics and general relativity but this is a really nice start down that road).

- Carlo ROVELLI: *Reality is not What It Seems. The Journey to Quantum Gravity*. New York: Penguin, 2016. (Translated by Simon Carnell and Erica Segre, original published in 2014).

There is no sign at all that Rovelli is to be seen as a strict finitist but I mention this book specifically for chapter 11, "The End of Infinity". A short chapter to show that physics is ultimately finite and that infinity has no place and even more so in the integrated quantum gravity theory. This obviously is not an elaborate argumentation, let alone a formal framework for strict finitism but as supporting idea it is nice.

- Marina CORTÊS & Lee SMOLIN: "Reversing the Irreversible: from limit cycles to emergent time symmetry". https://arxiv.org/abs/1703.09696 (uploaded 27 March 2017).

A quite intriguing paper as it suggests that an irreversible system can in the long run produce a reversible system on condition that the first system is discrete and finite. This produces a nice connection between the existence of a time's arrow that does not need to invoke special boundary conditions as these are "selected" by the underlying discrete system. This is, to be clear, one paper out of a continuous (sorry for the pun) production by Lee Smolin and collaborators. I have selected this paper because it is makes this link between the nature of time and the nature of (underlying) space.

- Stephen WOLFRAM: *A Class of Models with the Potential to Represent Fundamental Physics*. https://arxiv.org/ftp/arxiv/papers/2004/2004.08210.pdf (uploaded 15 April 2020).

This impressive book deals with a topic that the author knows better than anyone else – it is sufficient to have a look at the earlier book, A New Kind of Science (Champaign: Wolfram Media, 2002) – namely cellular automata. The major part of the book deals with that subject but the last chapter about the potential physical interpretations deserves our interest. Automata and graphs lead "naturally" to a discrete way of thinking and thus it leads to a form of physics, incorporating hodons and chronons. Although some detail is presented, a lot of work still needs to be done but then the author did indicate that we are talking about the potential. That being said, this is an important contribution to the idea that, one, graph-like structures are the basis to start from and, two, both space and time are derived concepts.

IV. Sources referring to computer science

- Zenon KULPA: "On the Properties of Discrete Circles, Rings, and Disks". *Computer Graphics and Image Processing* **10**, 1979, pp. 348-365.

In the opening paragraphs, the author states the following: "In the context of the applications of these objects it may be useful to investigate some of their formal properties. It is the purpose of this paper to take a step in this direction. First, discrete circles, disks, and rings are defined, then several of their properties are derived, namely, conditions for the occurrence of certain point configurations (ambiguity points and so-called "spikes"), formulas for the number of raster points in these objects, and their perimeters and areas. These parameters are also related to corresponding properties of ideal (nondiscrete) circles, and some limit theorems (for radius approaching infinity) are stated." This is again a fine example of formulating a discrete geometry without classical continuous geometry in the background. Although one would expect a number of such approaches, that is not really the case. Overall, the reference to the continuous case plays an important role and the discrete case therefore remains a 'derived' case.

- Elemér E. ROSINGER: "On the Safe Use of Inconsistent Mathematics". Url: http://arxiv.org/pdf/0811.2405v2.pdf, 2008. (This is an Arxiv-paper, as yet without a published counterpart.)

The starting point of this paper is that every computer embodies an inconsistent theory for, on the one hand, it is assumed that P(eano)A(rithmetic) holds and, on the other hand, a limit is accepted, that is, a number such that L = L + 1. It then goes on to select the largest consistent part of this combined inconsistent theory. That is a fairly classical approach but there are two interesting things. The first is that a distinction is made between the size of the numbers and the reasoning about those numbers. This seems to correspond well with the distinction between what is the case in the model and what can be expressed in the language. The second is that the limit is the least interesting part: "…it is made sure that additions, subtractions and multiplications lead to results well within the range of integers between −M and M." Reference is made to Chris Mortensen but the point is rather to avoid inconsistencies.

- Nils Anders DANIELSSON: *Axiomatic Discrete Geometry*. London: Imperial College, 2002. (Thesis submitted for MSc Degree in Advanced Computing.)

The first paragraph explains why this thesis is classified under computer science: "The main approaches to image analysis and manipulation, computational geometry, and related fields are based on continuous geometry. This easily leads to trouble with rounding errors and algorithms that return erroneous output, or even fail to terminate gracefully. In view of this we can argue that the proper framework for many algorithms is not continuous, but discrete. Furthermore it is preferable if such a framework is axiomatically defined, so that the essential properties of the system are clearly stated and many models can share the same theory." (p. 3) It refers to the work of Albrecht Hübler on discrete geometry and the thesis presents this approach in great detail (as Hübler's work is not easy to find). The core element to turn geometry into a discrete geometry is the axiom of discreteness: For any two points *p* and *p'* there is at most a finite number of points *q* such that *B*(*p; q; p'*), where *B* stands for the betweenness relation, familiar from standard axiomatisations of Euclidean geometry, Hilbert style. Apart from that Danielsson's approach relies mainly on projective geometry and matroids. If one were to have doubts about the possibility of a full axiomatisation of discrete geometry, this thesis makes clear that it can be done and it is mathematically quite interesting and challenging.

V. Philosophical discussions on strict finitism

- Patrick SCHINDLER: "Tense Logic for Discrete Future Time". *Journal of Symbolic Logic*, **35**(1), 1970, pp. 105-118.

Another example of the combination of tense logic and discrete time models. I will not include (at present) additional papers and books on this particular topic. Here it serves the purpose to show that this is a separate thread in the study of strict finitism. After all, if you want a strict finitist and applicable mathematics, space and time in physics will have to be discrete, one way or another. Thus it is a good idea to have some ideas about discrete space and time.

- Paul ZIFF: "The Number of English Sentences". *Foundations of Language*, **11**(4), 1974, pp. 519-532

A quite interesting article by Paul Ziff wherein he tries to refute the obvious answer to the question how many sentences can be expressed in English (or any other ordinary language for that matter), namely an infinite number. Although the argumentation is quite elaborate and philosophically deep, when applied to mathematics one of the important considerations he makes is the following. Suppose we write down the formula "If x > 0 then …", where the three dots represent some formula. In the framework of many a logic, if I can write down the above formula, I can also write down "If (x > 0 or x > 0) then …", where I use the brackets to avoid ambiguity. But once that is permitted, I can write down "If (x > 0 or x > 0 or ▪▪▪ or x > 0) then …", where the three dots ▪▪▪ stand for an arbitrary number of the formula "x > 0". But then the question is simply this: under what circumstances will a mathematician (or a philosopher for that matter), write down such an equation? Answer: never. Note that I myself did not in these comments. I used the "three-dot" mechanism.

- Kazimierz TRZESICKI: "Is discreteness of time necessary for Diodorean Master Argument?". *Bulletin of the Section of Logic*, **16**(3), 1987, pp. 125–131.

This short paper could just as well have been listed under logic, however logics like temporal or tense logic include discrete models as one of the possibilities without any reference to strict finitism. It is rather an attempt at being complete. However this paper claims that fatalism (in the Stoic sense) requires time to be discrete. This established a curious connection between the structure of time and a particular philosophical theory. It does not follow that therefore time is discrete but it is an unexpected argument in favour of it.

- Brian ROTMAN: "Toward a semiotics of mathematics". *Semiotica* **72**(1/2), 1988, pp. 1-35.
- Brian ROTMAN: *Signifying Nothing. The Semiotics of Zero*. Stanford: Stanford University Press, 1993.
- Brian ROTMAN: *Ad Infinitum ... The Ghost in Turing's Machine*. Stanford University Press, Stanford, 1993.
- Brian ROTMAN: "The Truth about Counting". *The Sciences*, **37**(6), 1997, pp. 34-39.
- Brian ROTMAN: *Mathematics as Sign. Writing, Imagining, Counting*. Stanford: Stanford University Press, 2000.

A separate place must be reserved for (part of) the work of Brian Rotman. Although, as far as I am aware, he does not consider himself a strict finitist. But, I assume that he is indeed critical of the infinite (and of zero as the book about zero indicates) and therefore interesting to a strict finitist. Furthermore in the 1993 *Ad Infinitum* book he proposes a formal model, called non-Euclidean arithmetics (so I could have included this book in the section on arithmetic as well, as there are some similarities with the work of Burgin discussed there, who talks about projective geometries). I will not present in full detail his approach but limit myself to a quote from a review that I wrote in 1996, "The strange case of the missing body of mathematics", of *Ad Infinitum* (*Semiotica*, **112**(3/4), 1996, pp. 403-413.):

> "In the last chapters of the book and in an appendix, Rotman gets well under way to develop and spell out in detail an arithmetic according to his model. I will not go into the technical aspects of this proposal that he labels "non-euclidean arithmetic". No explanation needed here, I guess. What is quite original about it, is that thermodynamical considerations enter into the picture. After all, why should entropy not affect mathematics *seen as an activity* as well? Perhaps the label "entropic or dissipative arithmetic" might do just as well."

More generally speaking, there are not that many philosophers, interested in mathematics, who (have) develop(ed) a *semiotic* perspective (and in Rotman's case to think about infinity). Another well-known example that comes to mind is Paul Ernest. Finally, it is worthwhile to consult his website: https://brianrotman.wordpress.com/.

- Amit HAGAR: *Discrete or Continuous? The Quest for Fundamental Length in Modern Physics*. Cambridge: Cambridge University Press, 2014.

This exciting book could be listed under a number of headings but I guess that philosophy is the best choice as the fundamental question discussed in the book is whether (physical) nature is discrete or continuous. What is really impressive is that it takes into consideration all the relevant physical theories so, at the same time, it is an introduction to quantum loop gravity, among many other things.

- James FRANKLIN: "Discrete and Continuous: A Fundamental Dichotomy in Mathematics". *Journal of Humanistic Mathematics*, **7**(2), July 2017, pp. 355-378.

Franklin is not a strict finitist but the issues that he deals with in this paper are obviously quite relevant for a strict finitist. After all, in a sense, strict finitists have a lot of explaining to do. If supposedly it (whatever 'it' is) is all strictly finite, where did the idea of the continuous come from and, more miraculously, how did we manage to reason with it, so successfully?

- Tyron GOLDSCHMIDT: "The Argument from (Natural) Numbers", in Jerry L. Walls and Trent Dougherty (eds.), *Two Dozen (or so) Arguments for God: The Plantinga Project* (Oxford: Oxford University Press, to appear 2017)

Finitism versus infinity apparently invites a divine connection. In this paper Goldschmidt develops the argument that since the number of human beings is finite and each human being can only think of a finite number of numbers, there must be a being that can think all of the natural numbers at once. This is one example of types of arguments that want to link the infinite with the divine. The upshot seems to be that therefore a strict finitist must be an atheist. I have to confess that I do not really see the connection. And do note that Goldschmidt believes strict finitism something impossible to believe.

- Bradley O. BASSLER: *Diagnosing Contemporary Philosophy with the Matrix Movies*. London: Palgrave, 2017.

A decidedly special book. It talks about philosophy, mathematics, physics, but also poetry and literature and uses the Matrix movies as background. Parts of the book deal with the work of Yessenin-Volpin but, even more prominently, the work of David Isles. Notions such as parafinite and paraphysics are introduced and discussed. Once again, there is no explicit mention of strict finitism or ultra-finitism, just the one reference to the ultra-intuitionism of Yessenin-Volpin on page 193. So, certainly not a defence or elaboration of strict finitism but a host of interesting philosophical thoughts related to the topic (including inevitably Ludwig Wittgenstein).

- Brent C. LYONS: “The Applicability of the Planck Length to Zeno, Kalam, and Creation Ex Nihilo”. *Philosophia Christi*, **19**(1), 2017, pp. 171-180.

A nice defense of discrete geometry, applied to the topics mentioned in the title of the paper: Zeno’s paradoxes (this covers familiar ground, see elsewhere in this bibliography), the cosmological Kalam argument (that is special, as it shows that philosophical-theological matters are related to the discrete-continuous debate), and the big bang as a transition from nothing to something. In addition, there is a subtle argumentation to show that epistemic considerations, that will be accepted by all, not exclusively strict finitists, impact metaphysical considerations. In short, to make the transition from “I perceive that the world is discrete” to “The world is discrete”.

- Jan HEYLEN: “Finitism versus infinitism: No 'truths' are lost”. In: Bart Van Kerkhove, Karen François, Steffen Ducheyne & Patrick Allo (eds.): *Laat ons niet ernstig blijven. Huldeboek voor Jean Paul Van Bendegem*. Gent: Academia Press, 2018, pp. 267-275.

This chapter by Jan Heylen presents important philosophical reflections on the notion of truth in a strict finitist framework (in casu, he looks specifically at my own strict finitist proposal, but the questions raised extend beyond that). His analysis leads to interesting considerations about the ontological commitments that go together with the use of natural numbers.

Walter DEAN: “Strict Finitism, Feasibility, and the Sorites”. *The Review of Symbolic Logic*, **11**(2), 2018, pp. 295-346.

This important and extensive paper takes the famous Wang’s paradox as starting point and discusses in great detail the relations between strict finitism and the feasibility and complexity of computations. Wang’s paradox is crucial in the sense that it creates a link between strict finitism and vagueness. The exemplar is the argument that, given the predicate “is small” and the variable n, ranging over natural numbers, if one accepts the premises, S(0) and S(n) → S(n+1), then, by mathematical induction, for all n, S(n), which sounds horribly wrong. In addition, there is an impressive list of references. It is worth remarking here that the discussion relating strict finitism, vagueness and feasibility is as good as a separate strand in the discussion about the possibility and coherence of strict finitism.

Interesting historical note: the label ‘Wang’s paradox’ is not related to any paper of Hao Wang but is used as the title of a paper by Michael Dummett [Michael DUMMETT: “Wang’s Paradox”. *Synthese* **30**, 1975, pp. 301-324]. At the end of that paper Dummett explains his choice of title: “The title relates to an article by Professor Hao Wang which I remember reading in an ephemeral Oxford publication many years ago. I should probably have abandoned it had I published the article sooner, since I never supposed that Professor Wang intended anything but to display the general form of a range of ancient paradoxes; but, since the name has gained some currency, I thought it better to leave it.” (pp. 323-324).

VI. My own contributions

- *Finite, Empirical Mathematics: Outline of a Model*. Werken uitgegeven door de Faculteit Letteren en Wijsbegeerte (Works edited by the Faculty of Arts and Letters), R.U.Gent, volume 174, Ghent, 1987. → short note about this book → pdf of the book

This is the shortened version, translated from Dutch into English, of my PhD, submitted and defended in 1983. The pdf is free to download and, of course, contains itself a host of references, up to that period. These have been integrated in this document. A request to anyone who wants to read the book: please read first the “short note about this book”. In one sentence: so many years later I am rather struck by my “naïveté” about the topic. Although I still cherish such ideas as “shemath” (the “sheet mathematician”) and the importance of the underlying logic.

- “Zeno’s Paradoxes and the Weyl Tile Argument”. *Philosophy of Science*, 1987, **54**(2), pp. 295-302.

A short paper where a solution is presented to the Weyl tile argument, namely that in a discrete geometry the hypothenuse of right triangle equals one of the sides. A summary is to be found in the entry in the *Stanford Encyclopedia of Philosophy* (see further in this listing).

- “Strict, Yet Rich Finitism”. In: Z.W. Wolkowski (ed.): *First International Symposium on Gödel’s Theorems*, World Scientific, Singapore, 1993, pp. 61-79.

This is the first of a set of papers that deal with a new approach to strict finitism based on paraconsistent logic. The main advantage is that this version, in contrast with my earlier presentation in my PhD (mentioned above), makes comparison with classical theories much easier.

- (editor), Modern Perspectives on the Philosophy of Space and Time. Special issue of *Philosophica*, **50**, 1993.

I was asked to be the editor of this volume. Contributors were:
- Graham PRIEST, On Time,
- John MCKIE, Transition and Contradiction,
- C. W. KILMISTER, Space, Time, Discreteness,
- Newton C. A. DA COSTA & Francisco Antonio DORIA, On the Incompleteness of Axiomatized Models for the Empirical Sciences.

Below is my own contribution to that volume.

- "How Infinities Cause Problems in Classical Physical Theories". *Philosophica* **50**, 1993, pp. 33-54.

My first exploration of strict finitism and physics, introducing supertasks, as this is a superb testcase for strict finitism. Full text: How Infinities Cause Problems in Classical Physical Theories.

- "Strict Finitism as a Viable Alternative in the Foundations of Mathematics". *Logique et Analyse*, 37(**145**), 1994 (date of publication: 1996), pp. 23-40.

A continuation of the Wolkowski paper, including both a technical presentation and a philosophical defense.

- "Ross' Paradox is an Impossible Super-Task". In: *The British Journal for the Philosophy of Science*, **45**, 1994, pp. 743-748.

My first publication on supertasks. This paper is a response to a previously published paper by Allis & Koetsier in the same journal.

- "In defence of discrete space and time". *Logique et Analyse*, 38(**150-151-152**), 1995 (date of publication: 1997), pp. 127-150.

Work on supertasks is continued in this paper but the focus is on strict finitist geometry, containing an answer to the question how a strict finitist can talk about circles, angles and rotations.

- "Ook het oneindige is ons werk". In: Diderik Batens (red.), *Leo Apostel. Tien filosofen getuigen*. Hadewijch, Antwerp/Baarn, 1996, pp. 119-134. (in Dutch)

This paper was written after Leo Apostel, my first teacher and mentor, passed away. It is more or less autobiographical and it explains how strict finitism became my research theme. It also mentions the *Signific Movement* and, more specifically, Gerrit Mannoury.

- *Tot in der eindigheid. Over wetenschap, New-Age en religie*. Hadewijch, Antwerp, 1997 (in Dutch).

I wrote this book because I had the idea that strict finitism should not be restricted to mathematics proper but should eventually be developed into a "full" philosophy of finitude. There is of course in philosophy a long tradition about the dichotomy between the finite and the infinite, often related to the man-God relation, especially if the deity has infinite characteristics such as being all-knowing. But my idea was to approach the matter from the perspective of analytic philosophy and not of "continental" philosophy. The title by the way is a play of words on the expression "Tot in der eeuwigheid", meaning "Until eternity", which I changed into its counterpart "Until finitude". The book, thanks to the publishing house, is freely available (in pdf) on my website: http://jeanpaulvanbendegem.be (and, if you are reading this document, then most likely, though not necessarily, you are on my website.)

- "Why the largest number imaginable is still a finite number". *Logique et Analyse*, **42**, 1999 (date of publication: 2002), pp. 107-126. → to download the paper

This paper addresses the specific question what, if there is a largest (natural) number, how we can reason and think about it. I have to confess I am still very fond of this paper as it led me to the idea that the largest number imaginable is that number about which no questions can be answered.

- "How to tell the continuous from the discrete". In: François Beets & Eric Gillet (réd.), *Logique en Perspective. Mélanges offerts à Paul Gochet*. Brussels: Ousia, 2000, pp. 501-511.

This short paper examines the question whether an experiment could be set up that allows one to test the hypothesis that the structure of space and time is discrete or not. The basic idea – and I found afterwards that I was not the first one to come up with this idea – is that the logistic function (or the so-called S-function) shows an entirely different behaviour according to the continuous or discrete representation of it. In the discrete case chaotic phenomena appear that are missing in the continuous case. Peter Forrest has explored similar ideas in one of his papers.

- "Inconsistencies in the history of mathematics: The case of infinitesimals". In: Joke Meheus (ed.): *Inconsistency in Science*. Dordrecht: Kluwer Academic Publishers, 2002, pp. 43-57 (Origins: Studies in the Sources of Scientific Creativity, volume 2).

In this paper I presented a model for infinitesimals from a strict finitist perspective. The basic idea was (and is) that, given a calculation consisting of a finite set of formulas (such as in the calculation of a derivative of a given function), involving both "standard" numbers (not to be understood in the non-standard analysis way) and infinitesimals, it is possible to make a distinction between them and hence to calculate meaningfully with infinitesimals in a strict finitist setting. It is not possible however to conclude from the statement "For every calculation, there exists infinitesimals such that …" that "There exist infinitesimals such that for every calculation …". The importance of this switch of quantifiers is, I guess, not unknown in logic. There are papers by Graham PRIEST and others that explore a paraconsistent road to infinitesimals and that can be very inspirational for strict finitists. These are the most important ones:
1. Graham PRIEST & Bryson BROWN: "Chunk and Permeate, a Paraconsistent Inference Strategy. Part I: The Infinitesimal Calculus". *Journal of Philosophical Logic*, **33**(4), 2004, pp. 379–388.
2. Graham PRIEST, R. BENHAM & C. MORTENSEN: "Chunk and Permeate III: the Dirac Delta Function". *Synthese*, 191, 2014, pp. 3057-62.
3. Graham PRIEST & Bryson BROWN: "Chunk and Permeate II: Bohr's Hydrogen Atom". *European Journal for the Philosophy of Science*, **5**(3), 2015, pp. 297-314.
4. David John SWEENEY: "Chunk and Permeate: The Infinitesimals of Isaac Newton". *History and Philosophy of Logic*, **35**(1), 2014, pp. 1-23.

- "Ontwerp voor een analytische filosofie van de eindigheid". *Algemeen Nederlands Tijdschrift voor Wijsbegeerte*, **95**(1), 2003, pp. 61-72. (In Dutch)

This is a further elaboration of what I started to do in "Tot in der eindigheid" (see above), namely to stress the importance of an analytical approach of what is means to be finite.

- "Die Grenzen der Mathematik sind die Grenzen ihrer Darstellbarkeit". In: Michael H.G. Hoffmann (Hrsg.), *Mathematik verstehen. Semiotische Perspektiven*. Hildesheim: Verlag Franzbecker, 2003, pp. 258-270.

This paper is based on a German translation of the review mentioned above (in the section on Brian Rotman): "The strange case of the missing body of mathematics", *Semiotica*, **112**(3/4), 1996, pp. 403-413.

- "Classical Arithmetic is Quite Unnatural". *Logic and Logical Philosophy*, **11**(11-12), 2003, pp. 231-249 (special issue: Proceedings of Logico-Philosophical Flemish-Polish Workshops II-IV). → pdf of the paper

In an attempt to make further links between strict finitism and classical arithmetic, the setup of a supertask is used. Before the end of the supertask, at every stage, arithmetic is strictly finite and has all kinds of nice properties but at the end itself it turns into classical arithmetic and more or less loses all these properties. The strength of this paper is more on the rhetorical, argumentative level than on the mathematics proper.

- "Non-Realism, Nominalism and Strict Finitism: The Sheer Complexity of It All". In: Pieranna Garavaso (ed.): *Philip Hugly and Charles Sayward, Arithmetic and Ontology*. Amsterdam: Rodopi, 2006, pp. 343-365. (Poznan Studies in the Philosophy of the Sciences and the Humanities, vol. 90).

This contribution is a response to the main paper in the book by Hugly and Sayward. Mainly philosophical and again an argument in favour of the complexity of strict finitism.

- "Een verdediging van het strikt finitisme". *Algemeen Nederlands Tijdschrift voor Wijsbegeerte*, **102**(3), 2010, pp. 164-183. (In Dutch)

This journal has once a year a topical issue. One author is asked to write the main paper and others are invited to comment and finally the author gets a chance to write a reply. In this case, I was asked to write the main article. Five reactions follow. Unfortunately, all in Dutch.

- "The Possibility of Discrete Time". In: Craig Callender (ed.), *The Oxford Handbook of Philosophy of Time*, Oxford, OxfordUniversity Press, 2011, pp. 145-162.

Strict finitism specifically focused on (the philosophy of) time. A kind of companion article to "Finitism in Geometry" (see below), where the focus is on space. Worth mentioning is this paper: Claudio MAZZOLA: "Can discrete time make continuous space look discrete?" *European Journal for Philosophy of Science*, **4**(1), 2014, pp. 19-30. This paper addresses the argument in the book chapter mentioned above (and I quote here from the abstract of the paper itself): "to the effect that, if time is discrete, then there should exist a correspondence between the motions of massive bodies and a discrete geometry. On this basis, he concludes that, even if space is continuous, it should nonetheless appear discrete. This paper examines the two possible ways of making sense of that correspondence, and shows that in neither case van Bendegem's conclusion logically follows." The paper shows, once more I am tempted to write, that thinking about strict finitism, discreteness (versus continuity), and so forth, is philosophically challenging and merits our attention.

- "A Defense of Strict Finitism". *Constructivist Foundations*, **7**(2), 2012, pp. 141-149.
  → A Defense of Strict Finitism

This paper is derived from the Dutch version mentioned above: "Een verdediging van het strikt finitisme." I still believe that in this paper the most important arguments pro and contra strict finitism have been brought together. The discussion is really on the philosophical level. Reference is made to concrete proposals but are mainly mentioned.

- “Finitism in Geometry”. *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.), url = http://plato.stanford.edu/entries/geometry-finitism/, The Metaphysics Research Lab at the Center for the Study of Language and Information, Stanford University, Stanford, CA, 2015.

The first version of this lemma in the SEF dates back to 2005. Given the SEF policy that every five years (upper limit, sooner is better) an update has to be made, the present version is the second update and the lemma keeps growing. To be honest, I am quite proud of it, as it shows that strict finitism in geometry, all too often ignored, is a mathematically and philosophically interesting topic. The bibliography is quite extensive and has also been integrated in this document.