Review article

The strange case of the missing body of mathematics*

JEAN PAUL VAN BENDEGEM

Every year, I teach a course on philosophy of science, including some philosophy of mathematics, to graduate students in science. Every year, I do a small test as to their comprehension of the (mathematical) infinite. I present them with the (in)famous Hilbert hotel.[1] There is this hotel, you see, with a denumerable number of rooms, each room labeled with a natural number. At the beginning of the story, the hotel is full, i.e., every room is occupied. Now, someone enters the hotel lobby and asks whether there is a room free. The manager says yes, for all he has to do is to ask everybody already in the hotel to move from room n to room n + 1. Thus room 1 becomes available. And so the story continues. If all goes well, at this stage, at least a couple of students will protest. They are quite convinced that there is something wrong with this story and that I am challenging them to find the mistake. A frequently given answer is that my definition of 'is full' is misleading, if not plainly wrong. If something is full, then nothing can be added to it, no matter what or how. Most puzzling of all, in addition, is the fact that if I ask these very same students to prove that the set of rationals is denumerable, they easily do so.[2]

If one is not able to resist the temptation to diagnose this situation, then, to quote Errett Bishop, what we have here is a rather strong case of *(mathematical) schizophrenia*:

> ... the following short list covers most of the ground: rejection of common sense in favor of formalism; debasement of meaning by wilful refusal to accommodate certain aspects of reality; inappropriateness of means to ends; the esoteric quality of the communication; and fragmentation. (Bishop in Rosenblatt 1985: 1)

What about the cure? If we ignore the possibility that the diagnosis is completely wrong, we do have an illness to cure. The answer seems quite trivial: restore the unity of the patient. The difficult part is: how? As far as I can see, there are two possible stratagems: (a) you explain to the

* Brian Rotman, *Ad Infinitum: The Ghost in Turing's Machine: Taking God Out of Mathematics and Putting the Body Back In*. Stanford, CA: Stanford University Press, 1993.

Semiotica 112–3/4 (1996), 403–413

0037–1998/96/0112–0403

patient that, what (s)he believes to be two irreconcilable individuals, is in fact a single individual, or (b) you try to convince the patient that the second personality does not exist, nothing but a fiction of his or her imagination. Strategy (a) is the major way of handling this problem today: any handbook will explain in full detail the counter-intuitive idea of infinity, yet, it will show that everything is all right. Yes, it is weird and it is crazy, but, no, it is not nonsense. Strategy (b), on the other hand, is not popular at all. In fact, only a small minority of (a few) philosophers and (some) mathematicians have been and are attracted to it. Among them, Brian Rotman, who, in two related books, has treated the *alpha* and *omega*[3] of mathematics: zero and infinity. Zero and what it signifies are treated in *Signifying Nothing: The Semiotics of Zero* (1993, a reprint of the 1987 edition). Infinity is the subject of the book under review.

Followers of strategy (b) are usually called *strict finitists* (henceforth, SF). The additional 'strict' is to distinguish this philosophical point of view from the Hilbertian idea of finitism that he formulated in connection with the problem of the consistency of arithmetic. Hilbert's finitism is restricted to the meta-level: all sign manipulation must be regulated by a finite set of finitely expressible rules. What the signs are about is of no importance (when talking about such things as consistency and the like[4]). What then, do SFs believe?

The easy answer to that question is to invite the reader to consult Ernst Welti's history of strict finitism (Welti 1987) or Annius Groenink's thesis (Groenink 1993) on more recent proposals.

The less easy, but better answer to that question is to simply state the beliefs themselves. Take the natural numbers as an example. Usually, one will present this in the form of a set $N = \{0, 1, 2, \ldots\}$. And, usually, it will be argued that there is no upper limit, for, given any natural number n, there is always a next one, namely $n+1$.[5] The SF will argue that, quite on the contrary, if we insist on the concrete and practical conditions that need to be satisfied in order to be able to execute that task, i.e., given n to write down $n+1$, then there will be a largest number, say L. A pure science-fiction version would run as follows:[6] suppose every elementary particle in the universe is used to write a digit on, and suppose we have written on every quark and lepton — real or virtual, no matter — a 9, then the next natural number does not exist. Not only that, on a more philosophical level, one could argue that what we have here is a Sorites-type argument:

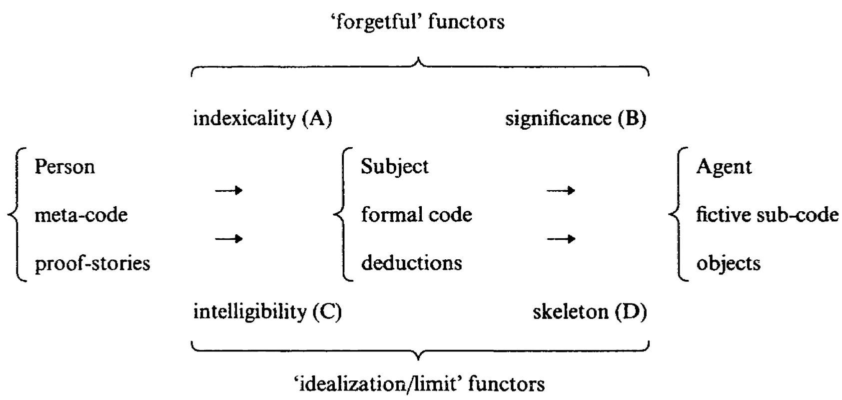
(premise)	I can write down 1
(premise)	If I can write down n, I can write down $n+1$
(conclusion)	I can write down every n.

Note that the conclusion follows because of mathematical induction, so to the mathematician the structure of the argument is sound. However, the conclusion is manifestly false; hence, since the first premise is perfectly acceptable, something must be wrong with the second one.[7]

Actually, at this stage, matters become rather subtle. I could be rightfully blamed for blurring a distinction: do I really mean that *the next natural number* does not exist? Or, should I mean that *the next numeral representing a natural number* does not exist? If it is the latter, then the question remains open whether that specific *number* exists or not. But then, strict finitism seems superfluous. For, although I have no name for that number in a specific language, viz., the decimal notation system, I do have other names for it, such as 'the first natural number that cannot be expressed as a numeral in the decimal notation system'.[8] Therefore, the strict finitist is forced to embrace the following slogan: *first there are numerals, then there are numbers, not the other way around.* What makes Rotman's contribution to the SF debate so valuable is that he takes a semiotician's point of view right from the start and continues to do so throughout the book. To him, it must be a matter of sign manipulation, of the relation between a sign, what it represents, and the interpreter, in other words, a full-scale semiotical analysis is what is needed and that is exactly Rotman's aim:[9]

> In what follows I offer a semiotic model of mathematics of the kind required, namely one that extends to the 'whole field of mathematical discourse'. I read mathematical signs in terms of a certain written practice, a business of manipulating inscriptions that characterizes mathematical thought as a kind of waking dream. (Rotman 1993 [1987]: xii)

What is this semiotic model? Rotman himself provides the reader with an excellent summary in the following diagram (Rotman 1993 [1987]: 92):



Let us start at the left side. The Person is the actual, real-life mathematician doing mathematics. The meta-code (s)he is using involves not only the mathematical symbols, but involves thought experiments as well (a crucial, if not an almost foundational category for Brian Rotman). It also involves trying out all sorts of ideas. In terms of proofs, what one arrives at are proof-stories or, if you like, proof-outlines. Eventually, however, if the proof-story holds good, it must be reformulated, perhaps even recreated, in the formal code, where the proof almost miraculously metamorphoses into a deduction. The end-result of this process is what we see in the mathematical journals, what we see presented on mathematical conferences, what most mathematicians in fact believe is what mathematics is all about. The author of a deduction cannot be the Person, for the latter will use expressions such as 'Suppose I take this set, and I do this or that operation, then with some neat trick, I can transform ...'. However, a deduction, for a start, lacks all reference to 'I'. Likewise, 'some neat trick' is an empty expression in a deductive context. So, who's doing this formal proof? It is the Subject.

This explains the role of the two arrows between Person and Subject. Arrow A, indexicality, is one of the 'forgetful' functors, since it erases, among other things, all reference to the 'I'. Arrow C, intelligibility, is one of the 'idealization' functors. Through the process of the elimination of person-specific properties, the Subject acquires the capability to perform certain acts that are excluded for the Person. A quote from Rotman himself will clarify immediately what is meant here:

> Standard mathematical practice stipulates (or, rather, implicitly assumes) the Subject's ideality to be such that he or she be free of error, misreadings, boredom, fatigue, memory loss, and misperception, insofar as these affect the reading/writing and manipulation of Code determined signs. (Rotman 1993 [1987]: 102)

The Subject, however, is still 'subject' to physical limitations. Hence, this cannot be the end of the story. When the Subject is writing down such imperatives as 'Repeat this operation *ad infinitum* and the following will result', then the Subject is imagining a further entity that is executing these tasks. Now this entity, whatever it is, exists, basically, within the text. This entity is what Rotman calls the Agent.[10] On the side of the 'forgetful' functors, the meaning of the arrow labeled 'significance' must be clear. The Agent does not know, in any sense of the word, what it is doing. What has now been erased from the mathematical picture, is (are) the meaning(s) of the signs involved. The Agent finds him/her/itself in the sign-world proper. If the Agent happens to be, say, a Turing machine, then it does, in effect, manipulate signs on a tape, it does move left and/or right and eventually, though not necessarily, it does stop.

Finally, the last arrow (D), one of the 'idealization' functors, to complete the diagram, is the skeleton arrow. In fact, this can be interpreted almost literally. When the Subject is no longer restrained by physical and/or biological limits, what we are left with is a skeleton. Perhaps not even that, for a real skeleton, too, will not withstand time running by. Rather, what is left 'will be a ghost: disembodied and unrelated to energy needs or spatio-temporal location or the effects of noise' (Rotman 1993 [1987]: 93).

To summarize in Rotman's own words,[11] what his model does is this (note that the use of the term 'prediction' does not commit Rotman to an empirical approach to mathematics, since we are talking here about *thought* experiments):

> Thus, in the course of a thought experiment whose aim is to prove some assertion/prediction X, the Person is an observer of the Subject's imagined manipulation of the Agent and, as a result of what s/he observes, is persuaded to accept X. (Rotman 1993 [1987]: 9)

In the last chapters of the book and in an appendix, Rotman gets well underway to develop and spell out in detail an arithmetic according to his model. I will not go into the technical aspects of this proposal, which he labels 'non-euclidean arithmetic'. No explanation is needed here, I guess. What is quite original about it is that thermodynamical considerations enter into the picture.[12] After all, why should entropy not affect mathematics *seen as an activity* as well? Perhaps the label 'entropic or dissipative arithmetic' might do just as well.[13]

The most attractive feature of Rotman's model and theory is that it outlines a quite 'natural' framework wherein mathematical symbols, mathematical proofs/texts, and real/ideal mathematicians find a place. In short, what we have here is the basis for a much-needed rhetoric of mathematics. For too long, the formalist ideal has kept us satisfied that, whatever mathematicians do, at the end of the day, it could be fitted into the formalist paradigm. But mathematical practice on its own is so diverse, so unexpected, and, sometimes, so bizarre, that it deserves a study on its own. I am not saying that Rotman is the first to draw our attention to this distorted situation,[14] but I do think he is the first to sketch a *global* picture.

Apparently Rotman knows the power of his proposal, for the strongest claim that he puts forward to support his approach is this:

The triadic scheme here suggests the basis for a unified critique of the three standard accounts of mathematics — namely, Brouwer's intuitionism (mathematics as Kantian constructions prior to any signs — 'languageless activity'); Hilbert's formalism (mathematics as game with uninterpreted symbols — 'meaningless marks on paper'); and Frege's Platonism (mathematics as discovery of truths, of thoughts 'timelessly true, true independently of whether anyone takes [them] to be true'). (Rotman 1993 [1987]: 83)

Here, I tend to side with Moore who wrote in his review of *Ad Infinitum*, the following:

First, it is hard to believe that such circumspection could ever catch on. For someone as concerned as Rotman is with the rhetorical power bases of actual mathematical practice, this is not a trivial point. He suggests at the end of his book that his conception may have the capacity to eclipse its rivals. But surely, unlike other revisionary philosophies of mathematics, it is too debilitating (and too much of an affront) for there to be any real historical prospect that the ordinary working mathematician will want to assimilate it. (Moore 1994: 24)

Why do I think that (unfortunately) I have to side with Moore? Quite simply, my own experience as a strict finitist tells me how hard and difficult it is just to get these views presented. One has every reason to believe that the (in)famous phrase of Hilbert — 'no one is going to chase us from the paradise that Cantor gave us' — and, especially, the word 'paradise' was meant in all seriousness.[15] Continuing the metaphor, this would mean that SFs are the atheists of mathematics. If so, would a religious person be impressed if an atheist told him or her that he has developed a framework wherein he, the atheist, can discuss God? I would guess not, for it does mean that God cannot be what the believer believes it to be, for the atheist did present a non-religious framework. Likewise, the Platonist will not be impressed, if the SF tells him or her that he can fit Platonism into his strict finitist framework.

At the same time, the problem is perhaps not as devastating as it seems on first sight. Given the 'natural' importance of the rhetorics of mathematics, it is obvious to expand the rhetorical view to the meta-level. In other words, what Rotman is doing is defending a proposal and trying to convince an audience that it is worthwhile 'buying it'. Hence, just as important is how the semiotic approach is presented, or, if you like, the rhetorics of rhetorics. For example, Moore, in the review discussed above, mentions two ways to present strict finitism. The first talks explicitly about (numerical) limits — it claims, e.g., that there is a largest, though finite, number equal to this or that — whereas the second one remains silent about the infinite.[16] I firmly believe that research into these different types of strict finitisms and, above all, to set up translations between them, is worthwhile not just for the technical fun of it, but for the implied rhetorical forceé.[17]

To summarize, there is no need to be extremely pessimistic about the success of strict finitism, but that does not justify, unfortunately, a victorious, optimistic view. The principal reason for this cautious attitude is my firm belief that concepts such as 'infinity', 'limits of thought', and 'expressibility', up to 'incomprehensibility', are deeply rooted in our western culture. There can be little doubt about this. Perhaps one might claim even more: it is a *key* to our culture. If in this review article I have made comparisons between mathematics and theology, this is not accidental at all. And if the reader thinks that it could perhaps be that long ago mathematicians did see a connection between the mathematical infinite and God's infinity, but that it could not be the case for modern man, let me try then a simple experiment. Here are two quotations. The question for the reader is quite simple: when were they written?

> This is one of the difficulties which arise when we attempt, with our finite minds, to discuss the infinite, assigning to it those properties which we give to the finite and limited; but this I think is wrong, for we cannot speak of infinite quantities as being the one greater or less than or equal to another.

> I have never proceeded from any 'Genus supremum' of the actual infinite. Quite the contrary, I have rigorously proven that there is absolutely no 'Genus supremum' of the actual infinite. What surpasses all that is finite and transfinite is no 'Genus'; it is the single, completely individual unity in which everything is included, which includes the 'Absolute', incomprehensible to human understanding. This is the 'Actus Purissimus' which by many is called 'God'.

My educated guess is that one is tempted to see the first quote as being recent, and the second one as perhaps late medieval, early renaissance, probably written by a mathematically inspired theologian. As a matter of fact, the first fragment was published in 1638, the second occurs in a letter written in 1908! The author of the former is Galileo Galilei,[18] the man who believed that the book of nature is written in the language of mathematics, the author of the latter Georg Cantor,[19] the founding father of modern set theory.

One isolated experiment does not make a case, but I am prepared to put forward the following claim. The claim is that, apparently, infinity is so tied up with religious and theological matters up to today, that to reflect on it necessarily means reflecting on the most famous triad of all: man-world-God. But any reflection on this triad implies an ethical stance, for it involves, at least, the question of the meaning of it all. Thus, the conclusion seems obvious: being a strict finitist is not just about doing mathematics and about doing physics, but, in the end (or at the very beginning) it is an ethical position. As every philosopher knows, ethical discussions, compared to ontological and epistemological discussions, are the hardest and toughest ones.

A final thought: it is perhaps funny that, although Rotman does quote Bishop (changing his first name to Everett) on several occasions, he did not mention his most famous remark, which must be exquisite music to Rotman's ears: 'If God has mathematics of his own that needs to be done, let him do it himself' (Bishop 1967: 2). What I like about this rash statement of Bishop is that it immediately raises the following difficult problem. If God does His own mathematics, then, I assume, we will have to do mathematics of *our* own. *If so, what are we supposed to do*? What Brian Rotman achieves in this work is to outline a framework wherein answers to that question can be properly formulated. A much-needed achievement, indeed.

Notes

1. This is just one of a series of examples. A deeply confusing example is Ross's paradox. Split up a time interval of one minute in an infinite denumerable series: $t_1=[0, 1/2]$, $t_2=[1/2, 1/4]$, ..., $t_n=[1/2^{n-1}, 1/2^n]$, ... Start with an empty urn and a denumerable amount of balls, labeled 1, 2, ..., n, ... During t_1, balls 1 to 10 go in the urn and 1 is taken back out; during t_2, balls 11 to 20 go in and 2 goes out, ..., during t_n, balls 10.(n-1)+1 to 10.n go in and n goes out, ... What is the state of the urn at the end of the minute? According to most, it should be empty! For, name any ball, say k, then this ball has been removed in the k-th time-interval. Or not? For a discussion of these matters, see Allis and Koetsier (1991) and Van Bendegem (1994).
2. I call this puzzling for what they do when they write down the proof, is precisely to formulate arguments that are completely analogous to the Hilbert hotel problem. Why are they troubled by the hotel but not by the proof?
3. My choice of words is not arbitrary, as will become clear in the sequel of this article.
4. I emphasize that Hilbert proposed this type of finitism in connection with the consistency problem. His view on the nature of mathematical knowledge, as put forward in his famous paper (1925) on the infinite, is closer to a kind of empirical interpretation, where we learn in practice that $2+3=3+2$, because we have been experimenting, say, with strokes or other discrete objects. We then generalize to statements such as $(\forall n)(\forall m)(n+m=m+n)$, thus introducing what Hilbert calls *ideal propositions*. To handle *this* type of proposition, Hilbert proposes his formalist philosophy.
5. In fact, it is even possible in the decimal notation system to formulate explicitly how to write down the next number:
(a) look at the digit at the right side,
(b0) if it is 0, change it to 1, and stop,
...
(b9) if it is 8, change it to 9, and stop,
(c) if it is 9, change it to 0. If there is a digit to the left, go to that digit and go to (b0). If there is no digit to the left, go to (d).
(d) Write a 1 right from the digit, and stop.

6. I am not claiming that SFs accept this kind of version. There is therefore no need to invalidate this story by raising quantum-mechanical or other similar objections.
7. Remarkably enough, there is a very deep connection between strict finitist views and the Sorites paradox. If one is quite serious about SF, then one should have a 'decent' reply to the Sorites problem. This is as difficult a task as one can see by an even superficial glance at the amount of literature on the subject today. A good start for a discussion on the link between SF and Sorites is Crispin Wright (1982).
8. Any reader familiar with the literature on the logical and mathematical paradoxes will recognize a variation on the Berry paradox. In short: even if I claim that a particular number has no name at all, then that description can be used as its name. If there is more than one number without a name, then we can quite simply order them and give the i-th element the name, the i-th number without a name. So, all numbers *do* have names? It depends: do I have enough names to order the unnameable numbers?
9. Rotman did the 'groundwork' for *Ad Infinitum* in a rather intriguing paper, 'Towards a semiotics of mathematics', written in 1980 and published in 1988 in this journal. Perhaps I should add here that, among the few reviews that have been devoted to Rotman's 1993 book, only Paul Ernest in *POME (Philosophy of Mathematics Education) Newsletter* 7, February 1994, is most explicit in his recognition of the fundamental contribution of Rotman to the field of semiotics, rather than to strict finitism, as a foundational theory in mathematics.
10. Perhaps one might wonder whether it is really necessary to have three actors in this play? Is the difference between Subject and Agent so large that we need to differentiate? I agree with Rotman that the answer must be yes. I will just give one rather extreme example, the so-called Cantor set, to illustrate the answer. Draw a line of a given length. Mathematics tells me that the number of points is of the order of the continuum. Divide the line in three parts and erase the middle part. What is left, is still of the order of the continuum. Divide each of the remaining parts in three and erase the middle parts. Repeat ad infinitum. On the one hand, there is a proof that the remaining set is still of the same order. On the other hand, it is perfectly impossible to make any drawing of this set, say, after a hundred stages. For the Person and the Subject, this Cantor set quite literally disappears out of sight. Only the Agent has it in focus.
11. There is a second summary that I myself tend to prefer but that seems more prone, I fear, to misinterpretation: 'The imagining Subject corresponds to the dreamer dreaming the dream, the skeleton Agent to the imago, the figure being dreamed, and the Person to the dreamer awake in the conscious subjectivity of language telling the dream' (Rotman 1993 [1987]: 9). The possible misinterpretation would be that, apparently, mathematicians could do well with a psychoanalytical treatment!
12. In a footnote, Rotman briefly mentions my own proposal for a strict finitist arithmetic (as presented in my 1987 book). In his terminology, what I did was to develop: 'a zero-dissipation model of resource depletion whose arithmetic would be a linearized projection of the non-Euclidean version to be constructed here' (Rotman 1993 [1987]: 183, note 21). What I did was to consider truncated versions of classical arithmetic. Thus there is a largest number L such that $L+1=L$. As said, I will not go into technical details, but, since the 1987 book, I have reformulated SF in terms of a paraconsistent logic — in short, a logic that does not accept the *ex falso*, thus, the presence of a contradiction does not lead to the collapse or trivialization of the system — quite different from what I did before. (See Van Bendegem 1992 and 1993 for details.)
13. Actually, when discussing the Agent, Rotman draws the analogy between the disembodied agent and Maxwell's demon. As one might recall, Maxwell's demon is an

imaginary creature that could overcome the famous law of entropy. To give a rough sketch: entropy says that a mixture of two types of gas, say type 1 and type 2, in a box will reach an equilibrium state and, almost certainly, remain there. Suppose now that in this state, we introduce in the box a screen with a little trap door that divides the box in two, say left and right. Thus we have the same mixture in both parts. What the demon does is this: if an atom of type 1 (2) in the left (right) side flies in the direction of the door, he opens it so that the atom enters in the right (left) side. Otherwise, he does nothing. What the demon does, in other words, is to separate the two types of gas. But that goes against the law of entropy, assuming, that is, that the demon does not consume energy.

14. See, e.g., as a starting point: Fischer, Restivo, and Van Bendegem (1993).
15. Seen from this angle, the following statement from Wittgenstein, rather than being somewhat bizarre, does acquire a deeper meaning: 'Imagine set theory's having been invented by a satirist as a kind of parody on mathematics. — Later a reasonable meaning was seen in it and it was incorporated into mathematics. (For if one person can see it as a paradise of mathematicians, why should not another see it as a joke?)' (Wittgenstein 1978: 264).
16. It is rather amusing to see age-old philosophical problems reappear in new contexts. The two forms proposed by Moore correspond, of course, to the well-known philosophical distinction between seeking limits from the outside and seeking limits from the inside.
17. This is the road I am exploring at the present moment, as mentioned in Note 12. The basic idea is to set up translations between strict finitist theories and classical infinite theories such that meta-theorems of the form 'If A holds classically, then it holds strict finitistically' can be proved, where the class of statements A is as large as possible. In fact, using a paraconsistent logic, this class covers all A.
18. See Galilei (1954): 31.
19. See Dauben (1979). This book still counts as the official biography of Georg Cantor. Dauben pays a lot of attention to Cantor's religious and theological ideas. Without these, some of his mathematical ideas are very hard, if not impossible, to understand. The fragment quoted is on page 290. The letter itself was addressed to Grace Chisholm Young, and is dated June 20, 1908.

References

Allis, V. and Koetsier, T. (1991). On some paradoxes of the infinite. *The British Journal for the Philosophy of Science* 42 (2), 187–194.

Auerbach, David (1994). Saying it with numerals. *Notre Dame Journal of Formal Logic* 35 (1), 130–146.

Bishop, Errett (1967). *Foundations of Constructive Analysis*. New York: McGraw-Hill.

Dauben, Warren Joseph (1979). *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Cambridge, MA: Harvard University Press.

Ernest, Paul (1994). Review of *Ad Infinitum ... The Ghost in Turing's Machine*, by Brian Rotman. *POME (Philosophy of Mathematics Education) Newsletter* 7, 19–21.

Fischer, Ronald; Restivo, Sal; and Van Bendegem, Jean Paul (eds.) (1993). *Math Worlds: New Directions in the Social Studies and Philosophy of Mathematics*. New York: State University of New York Press.

Galilei, Galileo (1954). *Dialogues Concerning Two New Sciences*, trans. by Henry Crew and Alfonso DeSalvio. New York: Dover Publications. [Originally published in 1638.]

Groenink, Annius V. (1993). *Uism and Short-sighted Models*. Unpublished Graduate Thesis, University of Utrecht.

Hilbert, David (1925). Über das Unendliche. *Mathematische Annalen* 95, 161–190.

—(1971): On the infinite, trans. by Stefan Bauer-Mengelberg. In *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*, Jean van Heijenoort (ed.), 367–392. Cambridge, MA: Harvard University Press.

Lavine, Shaughan (1994). *Understanding the Infinite*. Cambridge, MA: Harvard University Press.

Marion, Mathieu (1992). Wittgenstein and finitism. *Montréal: Cahiers du département de philosophie*.

Moore, A. W. (1994). Out for the count. *Times Literary Supplement* (July 1), 24.

Ratner, Sidney (1992). John Dewey, empiricism, and experimentalism in the recent philosophy of mathematics. *Journal of the History of Ideas*, 467–479.

Rosenblatt, Murray (ed.) (1985). *Errett Bishop: Reflections on Him and His Research*. Providence: American Mathematical Society.

Rotman, Brian (1988). Toward a semiotics of mathematics. *Semiotica* 72 (1/2), 1–35.

—(1993 [1987]). *Signifying Nothing: The Semiotics of Zero*. Stanford, CA: Stanford University Press. (Reprint of 1987 edition).

Van Bendegem, Jean Paul (1987). *Finite, Empirical Mathematics: Outline of a Model*. Ghent: Works published by the Faculteit Letteren en Wijsbegeerte, University of Ghent, 174.

—(1992). Strict finitism meant to please the anti-finitist. *ANPA-West* 3 (1), 6–17.

—(1993). Strict, yet rich finitism. In *First International Symposium on Gödel's Theorems*, Z. W. Wolkowski (ed.), 61–79. Singapore: World Scientific.

—(1994). Ross' paradox is an impossible supertask. *The British Journal for the Philosophy of Science* 45, 743–748.

Welti, Ernst (1987). *Die Philosophie des strikten Finitismus. Entwicklungstheoretische und mathematische Untersuchungen über Unendlichkeitsbegriffe in Ideengeschichte und heutiger Mathematik*. Bern: Peter Lang.

Wittgenstein, Ludwig (1978). *Remarks on the Foundations of Mathematics*, third edition. ed. by G. H. von Wright, R. Rhees, and G. E. M. Anscombe; trans. by G. E. M. Anscombe. Oxford: Basil Blackwell.

Wright, Crispin (1982). Strict finitism. *Synthese* 51, 203–282.

Jean Paul Van Bendegem (1953) is Professor of Logic and Philosophy of Science at the Vrije Universiteit Brussel (Belgium) 〈jpvbende@vnet3.vub.ac.be〉. His principal research interests are foundations of mathematics (strict finitism), foundations of physics (philosophy of space and time), and the relations between science and religion. He has published *Finite, Empirical Mathematics: Outline of a Model* (1987), *Theory and Experience* (1988), *Math Worlds: New Directions in the Social Studies and Philosophy of Mathematics* (1993), and 'Real-life mathematics versus ideal mathematics: The ugly truth' (1993).